

# IDENTITIES FOR THE LIE ALGEBRA $\mathfrak{gl}_2$ OVER AN INFINITE FIELD OF CHARACTERISTIC TWO

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**ABSTRACT.** In 1970 Vaughan-Lee established that over an infinite field of characteristic two the ideal  $T[\mathfrak{gl}_2]$  of all polynomial identities for the Lie algebra  $\mathfrak{gl}_2$  is not finitely generated as a T-ideal. But a generating set for this ideal of polynomial identities was not found. We establish some generating set for the T-ideal  $T[\mathfrak{gl}_2]$ .

**Keywords:** polynomial identities, PI-algebras, Lie algebras, minimal generating set, matrix algebra, Specht problem.

## 1. INTRODUCTION

We consider all vector spaces, algebras and modules over an infinite field  $\mathbb{F}$  of arbitrary characteristic  $p = \text{char } \mathbb{F}$  unless otherwise stated. Denote by  $\mathfrak{gl}_n$  be the Lie algebra of all  $n \times n$  matrices over  $\mathbb{F}$ . We write  $x_1 \cdots x_n$  for the left-normalized product  $(\cdots ((x_1 x_2) x_3) \cdots x_n)$  and define the adjoint  $\text{ad} x$  by  $y(\text{ad} x) = yx$ .

We start with basic definitions. Denote by  $\mathcal{L}(X)$  the free Lie algebra freely generated by  $x_1, x_2, \dots$ . Given a Lie algebra  $\mathcal{L}$ , an element  $f$  of the free Lie algebra  $\mathcal{L}(X)$  is called a *polynomial identity* for  $\mathcal{L}$  if  $f(a_1, \dots, a_k) = 0$  for all  $a_1, \dots, a_k$  from  $\mathcal{L}$ , where  $f(a_1, \dots, a_k)$  stands for the result of substitution  $x_i \rightarrow a_i$  in  $f$ . In other words, a polynomial identity is a universal formula that holds on the algebra. For short, polynomial identities are called identities. The ideal of all identities for  $\mathcal{L}$  is denoted by  $T[\mathcal{L}]$ . If a Lie algebra satisfies a non-trivial identity, then it is called a Lie PI-algebra. An ideal  $I \triangleleft \mathcal{L}(X)$  is called a *T-ideal* if  $I$  is stable with respect to all substitutions  $x_1 \rightarrow f_1, x_2 \rightarrow f_2, \dots$  for  $f_1, f_2, \dots \in \mathcal{L}(X)$ . Note that  $T[\mathcal{L}]$  is a T-ideal and any T-ideal  $I$  is equal to  $T[\mathcal{L}]$  for some  $\mathcal{L}$ . For example, we can take  $\mathcal{A} = \mathbb{F}\langle X \rangle / I$ . A T-ideal  $I$  is called *finitely based* if it is finitely generated as T-ideal.

In 1590 Specht posted the following problem for associative algebras: is the T-ideal of identities finitely based for every algebra? (Definitions can be naturally extended from the case of Lie algebras). In 1986 Kemer established a positive solution for this problem in case  $p = 0$ . On the other hand, in case  $p > 0$  there exists non-finitely based ideals of identities (see Belov, Grishin, Shchigolev, 1999).

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As about the case of Lie algebras, In 1970 Vaughan–Lee [9] established that over an arbitrary infinite field of the characteristic two there exists a finite dimensional algebra such that its T-ideal of identities is not finitely based. Namely, the T-ideal  $T[\mathfrak{gl}_2]$  is not finitely based, but a generating T-ideal  $T[\mathfrak{gl}_2]$  was not found. In 1974 Drensky [2] extended this result as follows. For an arbitrary infinite field of positive characteristic he constructed an example of finitely dimensional Lie algebra which T-ideal is not finitely based. In 1992 Il'tyakov [4] established that T-ideal of identities for any finite dimensional Lie algebra over a field of characteristic zero is finitely based. Note that the following question is still open: does any Lie algebra over a field of characteristic zero have a finitely based T-ideal of identities?

It is an interesting problem to explicitly describe a generating set for the T-ideal of identities of the particular Lie algebra. The identities for  $\mathfrak{gl}_2$  and  $\mathfrak{sl}_2$  were found by several mathematicians over any infinite field with the exception of identities for  $\mathfrak{gl}_2$  in the characteristic two case. Over a field of characteristic zero Razmyslov [6] found a finite set generating the T-ideal of identities for  $\mathfrak{sl}_2$ . A minimal generating set for the T-ideal  $T[\mathfrak{sl}_2]$  was established by Filippov [7]. Working over an infinite field of characteristic different from two Vasilovskii [8] showed that any identity for  $\mathfrak{sl}_2$  follows from the single identity

$$yz(tx)x + yx(zx)t = 0.$$

Over a field of characteristic different from two identities for  $\mathfrak{gl}_2$  coincide with identities for  $\mathfrak{sl}_2$ , since  $A - \frac{1}{2}\text{tr}(A)E$  satisfies every identity for  $\mathfrak{sl}_2$ , where  $A \in \mathfrak{gl}_2$ . It is not difficult to see that over an arbitrary (finite or infinite) field of characteristic two the ideal of identities for  $\mathfrak{sl}_2$  is generated by a single identity  $xyz$ . In 2009 Krasilnikov [5] showed that the T-ideal of the certain subalgebra of  $\mathfrak{gl}_3(\mathbb{Q})$  is finitely based.

We obtained an explicit description of identities (the proof is given at the end of the paper).

**Theorem 1.1.** *In the case of an infinite field  $\mathbb{F}$  of the characteristic two the T-ideal of identities for  $\mathfrak{gl}_2$  is generated by the following identities:*

- (a)  $(x_1x_2)(x_3x_4)x_5$ ;
- (b)  $(x_1x_2)(x_1x_2 \cdots x_k)$ , where  $k > 2$ ;
- (c)  $(x_1x_2)(x_3x_4) + (x_1, x_3)(x_2x_4) + (x_1x_4)(x_2x_3)$ ;
- (d)  $(x_1x_2)(x_3x_4 \cdots x_m) + (x_1x_3)(x_2x_4 \cdots x_m) + (x_1x_4)(x_2x_3 \cdots x_m)$ , where  $m > 4$ .

## 2. THE KNOWN RESULTS ON IDENTITIES OF $\mathfrak{gl}_2$

In the rest of this paper we assume that  $\mathbb{F}$  is an infinite field of the characteristic two. Denote  $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ . Given two  $2 \times 2$  matrices  $A$  and  $B$ , we write  $AB$  for the Lie product  $AB = [A, B] = A \cdot B - B \cdot A$ , where  $A \cdot B$  is the usual (associative) matrix multiplication.

Introduce some notations:

$$a = E_{22}, \quad b = E_{12}, \quad c = E_{21},$$

where  $E_{i,j}$  is  $2 \times 2$  matrix with the only non-zero entry in position  $(i, j)$ . Then  $a, b, c, bc = E_{11} + E_{22}$  is a base for  $\mathfrak{gl}_2$ . Note that

$$ba = b, \quad ca = c, \quad (bc)x = 0 \text{ for each } x \in \mathfrak{gl}_2.$$

Considering identities of  $\mathfrak{gl}_2$ , for short sometimes we write  $i$  instead of letter  $x_i$ . Moreover, we write  $12 \cdots k$  for  $(\cdots((12)3) \cdots)k$ . As an example, the Jacobi identity is  $123 + 231 + 312 = 0$ .

In this section we consider some facts obtained in [9].

**Lemma 2.1.** *The following identities hold in  $\mathfrak{gl}_2$ :*

- (1)  $(12)(34)5 = 0$ ;
- (2)  $(1 \cdots n)(12) = 0$ , where  $n \geq 2$ .

Denote by  $\mathcal{F}$  the quotient of the free Lie algebra with free generators  $y_1, y_2, \dots$  by the ideal generated by the identity  $(12)(34)5 = 0$ . Since  $(12)(34)5 = 0$  is an identity of  $\mathfrak{gl}_2$ , we can consider the T-ideal of identities of  $\mathfrak{gl}_2$  as a T-ideal in  $\mathcal{F}$ ; in this case we write down  $T[\mathfrak{gl}_2] \triangleleft \mathcal{F}$ . The algebra  $\mathcal{F}$  has  $\mathbb{N}_0$ -grading by the degrees and  $\mathbb{N}_0^n$ -grading by multidegrees. For short, we denote the multidegree  $(1, \dots, 1)$  ( $n$  times) by  $1^n$ . An element is called multilinear if it is  $\mathbb{N}_0^n$ -homogeneous and its degree with respect to each letter is either 0 or 1. Note that an element of multidegree  $1^n$  is always multilinear but the inverse statement does not hold.

**Lemma 2.2.** *The following identities hold in  $\mathcal{F}$ :*

- (1)  $(125)(34) = (12)(345)$ ;
- (2)  $(xy g_1 \cdots g_n)(uv) = (xy g_{\sigma(1)} \cdots g_{\sigma(r)})(uv g_{\sigma(r+1)} \cdots g_{\sigma(n)})$ , where  $n \geq 1$ ,  $0 \leq r \leq n$  and  $\sigma \in S_n$ .

**Lemma 2.3.** *The algebra  $\mathcal{F}^2$  satisfies the following properties:*

- (1)  $\mathcal{F}^2 = \mathbb{F}\text{-span}\{y_{i_1} \cdots y_{i_k} \mid k \geq 2, i_1 > i_2 \leq i_3\}$ ;
- (2)  $\mathcal{F}^2 = \mathbb{F}\text{-span}\{y_{i_1} \cdots y_{i_k} \mid k \geq 2, i_1 > i_2 \leq i_3 \leq \cdots \leq i_k\}$  modulo  $(\mathcal{F}^2)^2$ ;
- (3)  $(\mathcal{F}^2)^2$  is  $\mathbb{F}\text{-span}$  of  $(y_{i_1} \cdots y_{i_k})(y_{i_{k+1}} y_{i_{k+2}})$ , where  $k \geq 2$  and
  - $i_1 > i_2 \leq i_3 \leq \cdots \leq i_k$ ,
  - $i_{k+1} > i_{k+2} \leq i_3$ ,
  - $i_2 \geq i_{k+2}$ ,
  - $i_1 \geq i_{k+1}$  in case  $i_2 = i_{k+2}$ .
- (4)  $(\mathcal{F}^2)^3 = 0$ .

**Lemma 2.4.**

- (1) *The element  $(y_1 \cdots y_n)(y_1 y_2)$  with  $n \geq 3$  does not belong to the T-ideal, generated by  $\{(1 \cdots k)(12) \mid k \geq 3, k \neq n\}$  in  $\mathcal{F}$ . In particular,  $(y_1 \cdots y_n)(y_1 y_2) \neq 0$  in  $\mathcal{F}$ .*

- (2) The result of the complete or a partial linearization of  $(y_1 \cdots y_n)(y_1 y_2)$  in  $\mathcal{F}$  is zero.
- (3) For every non-zero identity  $f \in T[\text{gl}_2] \triangleleft \mathcal{F}$  we have  $f \in (\mathcal{F}^2)^2$ .
- (4) Assume that a non-zero identity  $f \in T[\text{gl}_2] \triangleleft \mathcal{F}$  is not multilinear. Then  $f$  an element of the  $T$ -ideal of  $\mathcal{F}$  generated by  $(1 \cdots n)(12)$ , where  $n \geq 2$ .

### 3. GENERATING SET

We use notations and conventions from Section 2. For short, we denote a product  $n(n+1) \cdots (s-1)(s+1) \cdots t$  by  $n \cdots \hat{s} \cdots t$ , where  $r \leq s \leq t$ .

**Lemma 3.1.** *Every identity  $f \in T[\text{gl}_2] \triangleleft \mathcal{F}$  of multidegree  $1^n$  can be represented as the following sum:*

$$(1) \quad f = \sum_{4 \leq i \leq n} \alpha_{i2}(i34 \cdots \hat{i} \cdots n)(21) + \sum_{3 \leq i \neq j \leq n} \alpha_{ij}(i23 \cdots \hat{i} \cdots \hat{j} \cdots n)(j1),$$

where  $\alpha_{ij} \in \mathbb{F}$ . In particular,  $n \geq 4$ .

*Proof.* We work in  $\mathcal{F}$ . We start with the case of an element  $w = (i_1 \cdots i_k)(pq)$  of multidegree  $1^n$  that satisfies conditions from part 3 of Lemma 2.3, namely,  $k \geq 2$ ,  $i_1 > i_2 \leq i_3 \cdots \leq i_k$ ,  $p > q \leq i_3$ ,  $i_2 \geq q$ . Hence  $q$  is the least element of  $\{1, \dots, n\}$ , i.e.,  $q = 1$ . Thus  $w = (i_1 \cdots i_k)(p1)$ , where  $\{i_1, \dots, i_k, p\} = \{2, \dots, n\}$  and  $i_1 > i_2 \leq i_3 \cdots \leq i_k$ . Moreover,

- if  $p = 2$ , then  $w = (i34 \cdots \hat{i} \cdots n)(21)$  for some  $4 \leq i \leq n$ ;
- if  $3 \leq p \leq n$ , then  $w = (i23 \cdots \hat{i} \cdots \hat{j} \cdots n)(j1)$  for some  $3 \leq i \leq n$ ,  $i \neq p$ .

Consider an element  $f$  from the formulation of lemma. By part 3 of Lemma 2.4 we have that  $f \in (\mathcal{F}^2)^2$ . Part 3 of Lemma 2.3 together with the above reasoning completes the proof.  $\square$

Given  $f = f(x_1, \dots, x_n) \in \mathcal{F}$ , denote by  $f^{(i,j)}$  the result of the following substitution in  $f$ :  $x_i \rightarrow b$ ,  $x_j \rightarrow c$  and  $x_k \rightarrow a$  for all  $1 \leq k \leq n$  with  $k \neq i, j$ . It is easy to verify the following claim.

**Lemma 3.2.** *An element  $f \in \mathcal{F}$  given by formula (1) is an identity for  $\text{gl}_2$  if and only if  $f^{(i,j)} = 0$  for all  $1 \leq i < j \leq n$ .*

**Lemma 3.3.** *An element  $f \in \mathcal{F}$  given by formula (1) is an identity for  $\text{gl}_2$  if and only if the coefficients  $\{\alpha_{ij}\}$  satisfy the following conditions:*

$$(2) \quad \alpha_{sr} = \alpha_{rs} \quad \text{for all} \quad 3 \leq s < r \leq n;$$

$$(3) \quad \alpha_{r2} = \sum_{3 \leq j \neq r \leq n} \alpha_{rj} \quad \text{for all} \quad 4 \leq r \leq n.$$

*Proof.* By the condition of Lemma 3.1 we have that  $n \geq 4$ . Note that it could be more convenient for the reader to consider the case of  $n = 4$  separately (see Example 3.4 below).

Assume that  $3 \leq s < r \leq n$ . Then the equality  $f^{(s,r)} = 0$  is equivalent to (2).

Assume that  $4 \leq r \leq n$ . Then the equality  $f^{(1,r)} = 0$  is equivalent to (3). The equality  $f^{(2,r)} = 0$  is equivalent to

$$\alpha_{r2} + \sum_{3 \leq i \neq r \leq n} \alpha_{ir} = 0,$$

which is a linear combination of (2) and (3).

The equality  $f^{(1,3)} = 0$  is equivalent to

$$(4) \quad \sum_{4 \leq i \leq n} \alpha_{i2} + \sum_{3 < j \leq n} \alpha_{3j} = 0.$$

Denote the second sum of (4) by  $A$ . Applying (3) to  $\alpha_{i2}$  we obtain that (4) is a linear combination of (3) and  $\sum \alpha_{ij} = A$ , where the sum ranges over all  $4 \leq i \leq n$ ,  $3 \leq j \leq n$  with  $i \neq j$ . The left hand side of this equality is equal to

$$\sum_{4 \leq i \leq n} \alpha_{i3} + \sum_{4 \leq i \neq j \leq n} \alpha_{ij} = \text{/see (2)/} = A + 2 \sum_{4 \leq i < j \leq n} \alpha_{ij}.$$

Thus (4) is a linear combination of (2) and (3).

The equality  $f^{(2,3)} = 0$  is equivalent to

$$\sum_{4 \leq i \leq n} \alpha_{i2} + \sum_{3 < i \leq n} \alpha_{i3} = 0,$$

which is a linear combination of (2) and (4).

The equality  $f^{(1,2)} = 0$  is equivalent to

$$\sum_{3 \leq i \neq j \leq n} \alpha_{ij} = 0.$$

Since the left hand side of this equality is

$$\sum_{3 \leq i < j \leq n} (\alpha_{ij} + \alpha_{ji})$$

it follows from (2). The claim is proven.  $\square$

For  $n \geq 4$  denote

$$f_n = (12)(34 \cdots n) + (13)(24 \cdots n) + (14)(23 \cdots n) \in \mathcal{F}.$$

In particular,  $f_4 = (12)(34) + (13)(24) + (14)(23)$ .

**Example 3.4.** In this example we illustrate the proof of Lemma 3.3 in case  $n = 4$ . Assume that  $0 \neq f \in \mathcal{F}$  is an identity for  $\mathfrak{gl}_2$  of multidegree  $1^4$ . By Lemma 3.1,  $f = \alpha_{42}(43)(21) + \alpha_{43}(42)(31) + \alpha_{34}(32)(41)$ . Since  $f^{(1,2)} = 0$ , then  $\alpha_{43} + \alpha_{34} = 0$ . Since  $f^{(1,3)} = 0$ , then  $\alpha_{42} + \alpha_{34} = 0$ . Since  $f^{(1,4)} = 0$ , then  $\alpha_{42} + \alpha_{43} = 0$ . Thus, the coefficients  $\{\alpha_{ij}\}$  satisfy

conditions (2) and (3) for  $n = 4$ . Moreover  $f = \alpha_{42}f_4$ . By straightforward computations (or applying Lemma 3.2) we can verify that  $f_4$  is actually an identity for  $\mathfrak{gl}_2$ . Therefore modulo multiplication by a constant,  $f = f_4$ .

**Lemma 3.5.** *Every identity  $f \in T[\mathfrak{gl}_2] \triangleleft \mathcal{F}$  of multidegree  $1^n$  is a linear combination of identities  $f_n(x_{\sigma(1)}, \dots, x_{\sigma(n)}) = 0$  for  $n \geq 4$  and  $\sigma \in S_n$ .*

*Proof.* By Lemma 3.1, we have that  $f$  is given by formula (1) for some coefficients  $\alpha_{ij} \in \mathbb{F}$ . It is convenient to form the vector  $\underline{\alpha} = (\alpha_{r2}, \alpha_{ij} \mid 4 \leq r \leq n, 3 \leq i \neq j \leq n)$ . Since  $f$  is uniquely determined by  $\underline{\alpha}$ , we denote  $f$  by  $f_{\underline{\alpha}}$ . By Lemma 3.3, coefficients  $\{\alpha_{ij}\}$  satisfy conditions (2) and (3). Hence coefficients  $\{\alpha_{sr} \mid 3 \leq s < r \leq n\}$  are “free” variables and the rest of coefficients are linear combinations of them. If  $\alpha_{sr} = 1$  for some  $3 \leq s < r \leq n$  and  $\alpha_{ij} = 0$  for all  $3 \leq i < j \leq n$  with  $(i, j) \neq (s, r)$ , then we denote  $f_{\underline{\alpha}}$  by  $f_{(sr)}$ . Obviously, for each  $\gamma, \delta \in \mathbb{F}$ , we have  $\gamma f_{\underline{\alpha}} + \delta f_{\underline{\beta}} = f_{\gamma \underline{\alpha} + \delta \underline{\beta}}$ . Thus any identity  $f_{\underline{\alpha}}$  is a linear combination of identities  $f_{(sr)}$  with  $3 \leq s < r \leq n$ .

To compute  $f_{(pq)}$  for  $1 \leq p < q \leq n$  we point out that  $f_{(pq)} = f_{\underline{\alpha}}$ , where the only non-zero coefficients from  $\{\alpha_{ij}\}$  are

- $\alpha_{pq} = \alpha_{p2} = \alpha_{q2} = 1$  in case  $p \neq 4$ ;
- $\alpha_{3q} = \alpha_{q2} = 1$  in case  $p = 3$ ;

Assume  $p = 3$ . Then

$$f_{(pq)} = (q34 \cdots \hat{q} \cdots n)(21) + (324 \cdots \hat{q} \cdots n)(q1) + (q24 \cdots \hat{q} \cdots n)(31).$$

For this proof it is convenient to rewrite  $f_n$  in  $\mathcal{F}$  as

$$(435 \cdots n)(21) + (325 \cdots n)(41) + (425 \cdots n)(31).$$

In case  $q = 4$  we have  $f_{(pq)} = f_n$  in  $\mathcal{F}$ . In case  $q \geq 5$  the result of substitution  $x_4 \rightarrow x_q$ ,  $x_q \rightarrow x_4$  in  $f_n$  is equal to  $f_{(pq)}$  in  $\mathcal{F}$ , because the identity from part (2) of Lemma 2.2 holds in  $\mathcal{F}$ .

Assume  $p \neq 3$ . Then

$$\begin{aligned} f_{(pq)} &= (p34 \cdots \hat{p} \cdots n)(21) + (q34 \cdots \hat{q} \cdots n)(21) + \\ &+ (p23 \cdots \hat{p} \cdots \hat{q} \cdots n)(q1) + (q23 \cdots \hat{p} \cdots \hat{q} \cdots n)(p1). \end{aligned}$$

Denote by  $h$  the result of substitution  $x_3 \rightarrow x_q$ ,  $x_q \rightarrow x_3$  in  $f_n$ . Applying identity from part (2) of Lemma 2.2 we obtain that

$$h = (4q35 \cdots \hat{q} \cdots n)(21) + (q235 \cdots \hat{q} \cdots n)(q1) + (4235 \cdots \hat{q} \cdots n)(q1).$$

In case  $p = 4$  we have that

$$f_{(pq)} + h = (435 \cdots n)(21) + (q345 \cdots \hat{q} \cdots n)(21) + (4q25 \cdots \hat{q} \cdots n)(21).$$

Applying part (2) of Lemma 2.2 we can rewrite the first summand as

$$(34q5 \cdots \hat{q} \cdots n)(21) = (4q35 \cdots \hat{q} \cdots n)(21) + (q345 \cdots \hat{q} \cdots n)(21)$$

(see also the Jacobi identity). Thus  $f + h = 0$  in  $\mathcal{F}$  and the claim is proven.

Assume  $p \geq 5$ . Denote by  $g$  the result of substitution  $x_4 \rightarrow x_p$ ,  $x_p \rightarrow x_4$  in  $f_n$ . Applying identity from part (2) of Lemma 2.2 we obtain that  $g$  is equal to

$$(pq34 \cdots \hat{p} \cdots \hat{q} \cdots n)(21) + (q234 \cdots \hat{p} \cdots \hat{q} \cdots n)(p1) + (p234 \cdots \hat{p} \cdots \hat{q} \cdots n)(q1).$$

Then  $f_{(pq)} + g$  is equal to

$$(p34 \cdots \hat{p} \cdots n)(21) + (q34 \cdots \hat{q} \cdots n)(21) + (pq34 \cdots \hat{p} \cdots \hat{q} \cdots n)(21).$$

Applying part (2) of Lemma 2.2 to the first summand and the second summand we obtain  $(3pq4 \cdots \hat{p} \cdots \hat{q} \cdots n)(21)$  and  $(q3p4 \cdots \hat{p} \cdots \hat{q} \cdots n)(21)$ , respectively. Then the Jacobi identity implies that  $f_{(pq)} + g = 0$ . The claim is proven.  $\square$

Now we can proof Theorem 1.1.

*Proof.* At the end of paper [9] it is shown that the T-ideal  $T[\mathfrak{gl}_2] \triangleleft \mathcal{F}$  is generated by

- some multilinear identities;
- identities  $(12)(12 \cdots n)$  for all  $n \geq 3$ .

Lemma 3.5 and the definition of  $\mathcal{F}$  together with part (1) of Lemma 2.1 conclude the proof.  $\square$

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